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The Heuristic Power of Representation in Descartes: Reflections on its Role in Thermodynamics

El poder heurístico de la representación en Descartes: Reflexiones sobre su papel en la Termodinámica.

O poder heurístico da representação em Descartes: Reflexões sobre seu papel na termodinâmica.

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Abstract.

In numerous scholarly works, we emphasize the significance and centrality of the concept of representation, along with its heuristic function, in advancing scientific progress. This study aims to explore the origins of this concept in Descartes' method, which linked geometry to autonomous mechanisms and subsequently translated these connections into algebraic language through the development of analytical geometry. By recognizing autonomous machines as a fundamental metaphor within Cartesianism, the epistemological notion of representation is decomposed into language and program, thereby acquiring an operational dimension. Through a revisitation of Carnot's theory, this work examines how he formulated a representation to address the problem of maximizing the efficiency of heat engines and how this representation evolved over subsequent decades. These developments led to the emergence of representations with progressively greater heuristic power, revealing a striking parallel with Cartesian design principles. This parallel serves to particularize Duhem's thesis on the history of physics, which posits that physical theories evolve to represent experimental laws through structures of increasing heuristic efficacy.

Keywords: Autonomous Machine. Mechanized Geometry. Analytical Geometry. Representation and Heuristic Power. Representations of Thermodynamics.

Resumen.

En numerosos trabajos académicos, destacamos la importancia y la centralidad del concepto de representación, junto con su función heurística, para el avance del progreso científico. Este estudio busca explorar los orígenes de este concepto en el método de Descartes, quien vinculó la geometría con los mecanismos autónomos y posteriormente tradujo estas conexiones al lenguaje algebraico mediante el desarrollo de la geometría analítica. Al reconocer las máquinas autónomas como una metáfora fundamental dentro del cartesianismo, la noción epistemológica de representación se descompone en lenguaje y programa, adquiriendo así una dimensión operativa. A través de una revisión de la teoría de Carnot, este trabajo examina cómo se formuló una representación para abordar el problema de maximizar la eficiencia de las máquinas térmicas y cómo esta representación evolucionó en las décadas posteriores. Estos desarrollos condujeron al surgimiento de representaciones con un poder heurístico progresivamente mayor, revelando un sorprendente paralelismo con los principios de diseño cartesianos. Este paralelismo sirve para particularizar la tesis de Duhem sobre la historia de la física, que postula que las teorías físicas evolucionan para representar leyes experimentales mediante estructuras de eficacia heurística creciente.

Palabras clave: Máquina Autónoma. Geometría mecanizada. Geometría analítica. Representación y poder heurístico. Representaciones de la termodinámica.

Resumo.

Em numerosos trabalhos acadêmicos, enfatizamos a importância e a centralidade do conceito de representação, juntamente com sua função heurística, no avanço do progresso científico. Este estudo visa explorar as origens desse conceito no método de Descartes, que vinculou a geometria a mecanismos autônomos e, posteriormente, traduziu essas conexões para a linguagem algébrica por meio do desenvolvimento da geometria analítica. Ao reconhecer as máquinas autônomas como uma metáfora fundamental dentro do cartesianismo, a noção epistemológica de representação é decomposta em linguagem e programa, adquirindo assim uma dimensão operacional. Por meio de uma revisitação da teoria de Carnot, este trabalho examina como ele formulou uma representação para abordar o problema de maximizar a eficiência de máquinas térmicas e como essa representação evoluiu nas décadas subsequentes. Esses desenvolvimentos levaram ao surgimento de representações com poder heurístico progressivamente maior, revelando um paralelo marcante com os princípios de projeto cartesianos. Esse paralelo serve para particularizar a tese de Duhem sobre a história da física, que postula que as teorias físicas evoluem para representar leis experimentais por meio de estruturas de eficácia heurística crescente.

Palavras-chave: Máquina Autônoma. Geometria Mecanizada. Geometria Analítica. Representação e Poder Heurístico. Representações da Termodinâmica.

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1. Introduction

The modern conception of representation can be traced back to a pivotal figure in science who utilized the power of reason to move Western culture from a state of skepticism to one of scientific optimism (Gilson, 1950, p. 126; Koyré, 1963). Representation, in Descartes, corresponds to a complex notion that articulates elements of his mechanistic view of reality with his epistemology, according to which human knowledge does not catch reality in itself. Our ideas are representations of reality. However, like almost everything in René Descartes (1596-1650), these theses are born and organized around mathematical questions.

2. The knowledge of reality by Descartes

There are several ways to present the Cartesian idea of representation. We want to do it in connection with a dispute that seems to be a mere terminological question, namely, how to classify the mathematical curves. This discussion marks the beginning of an objective area of research that gradually evolves into the mechanistic project. In this context, other elements of Cartesianism, such as the epistemological role of machines and the synthesis of analytical geometry, emerge as subsequent advancements that expand this area of research both phenomenologically and heuristically. However, before delving into these aspects, it is advisable to first consider the nature of the knowledge associated with the *Cogito*.

In the *Meditations on First Philosophy*, a work from Descartes' mature phase, fundamental epistemological questions are addressed in a foundational manner. This text not only explores how reason operates but also justifies and legitimizes these processes. Thinking is synonymous with being aware. Phrases like 'thinking about something' and 'being aware of something' essentially mean 'having an idea about something'. The idea, as a form of thought, enables the subject to become aware of their actions and to recognize themselves as the agent of their conscious acts. Additionally, the idea serves as an image of things, presenting specific content to the consciousness of the subject. In this sense, the idea acts as a representation of things or objects (Descartes, AT, VII, pp. 23-34)[1].

What is immediately available to thought are representations. The objects that representations present to consciousness exist independently of their formal existence as such (in themselves). Access to these objects is mediated through representations. This is due to the influence of methodical doubt, which separates the represented objects from the reality of the objects themselves. Consequently, the certainty of the act of representation does not guarantee the certainty of realities external to consciousness.

Since external objects are accessible only through the mediation of representations, it is impossible to compare these objects directly with their representations, as access to them is inherently mediated. Given the unfeasibility of direct access, the issue of the truth of representations must be addressed by relying exclusively on the intrinsic and immanent properties of representations that enable the recognition of their correspondence with external objects. The primary epistemological question is not how the idea is initially posited as a representation of external bodies but rather how this representation can be deemed clear and distinct, and thus, objectively valid (Gueroult, 1953, p. 141). These properties are encapsulated in a criterion or general rule of truth, articulated through the clarity and distinctness of ideas. This general rule allows us to evaluate ideas as true or false. Without this criterion, all ideas would be considered equal in terms of truth.

However, it is only in judgments that the connection between ideas and the things they represent is formally established. For these judgments to be considered indubitable, the ideas that compose them must be clear and distinct. While challenges exist regarding these concepts, they form the foundation for the criterion of truth, based on the inherent properties of ideas. Moreover, there are no alternatives within Cartesianism, as the *Cogito* does not dispel the obscurity that methodical doubt casts over the horizon of knowledge. The certainty derived from the *Cogito* stands as an exception, lacking the sufficient force to substantially alter this state.

The formulation of the criterion of truth emerges from the analysis of the *Cogito*, where the statement "I think, therefore I am" is revealed to be true and certain, provided it is understood clearly and distinctly by the *Cogito*. However, the justification or legitimization of this criterion is achieved only through the intervention of the transcendent Absolute. While metaphysical doubt serves as a strategy to compel reason to justify its operations, this step occurs only after the existence of God has

1 In referencing Descartes' works, we use the edition compiled by Charles Adam and Paul Tannery (commonly abbreviated as AT).

been established. Thus, the clarity and distinctness of ideas are validated as a criterion of truth (Landim Filho, 1992, pp. 121-6).

Beyond the *Cogito*, the criterion of truth facilitates the recognition of fundamental concepts in geometry, such as points, straight lines, and planes, which underpin Euclidean geometry. In physics, it enables the reduction of objects and their phenomena to concepts of extension and motion (Descartes, AT, VII, p. 63). The truth of these fundamental notions, intuitively apprehended as clear and distinct, can be conveyed through complex theoretical systems, provided that the connections between related propositions – such as premises and conclusions in logical deductions – are sufficiently robust to justify this transfer. The nature of these connections is as crucial to the construction of science as the clarity and distinctness of ideas, which lead to certain and true knowledge.

3. The mechanization of geometry or geometrization of mechanics

Descartes conducts a critical examination of the geometry developed by the ancients. He expresses perplexity regarding their distinction between two categories of curves: the mechanical and the geometric. While the first ones were those necessitating the assistance of some mechanical device to be drawn, the second ones were those fundamental to an established repertoire of proofs in geometry. The difference would lie in the need or not of an instrument (machine) to draw them.

For Descartes, the significance of this question does not reside in the distinction itself but in the underlying assumptions that sustain it. While geometric curves are regarded as exact, mechanical curves are considered imprecise. Consequently, the mechanisms that produce mechanical curves are deemed incapable of ensuring exactness. When this critique is extended beyond the realm of geometry, it results in a broader devaluation of machines. Descartes' critique of this distinction stems from his view that the ancients had undervalued machines, particularly their ability to produce precision and accuracy. From a Cartesian perspective, machines represent the most effective instruments available to humanity for achieving such exactness.

Descartes reinterprets the significance of the distinction between geometric and mechanical curves, arguing that the use of machines should encompass even the straight lines and circumferences produced by rulers and compasses, which he considers basic machines (Descartes, AT, VI, p. 381). However, he acknowledges that not all machines are capable of ensuring exactness. Consequently, rather than rejecting the traditional classification of curves, he refines it by specifying the types of machines that can reliably preserve accuracy. If it introduces inaccuracies, it will not be useful. Therefore, the distinction of the ancients must be re-elaborated using the Cartesian idea of machine, which can confer accuracy to curves generated through it.

These machines are not physical. Although composed of specific parts and couplings, they are pure extensions that move orderly. They constitute, in reality, a machine idea, operating in such a way as to mechanize geometry. This is the mechanized geometry, of which *Dioptrics*, an essay of the *Discourse of the Method*, can be seen as an application (Burnett, 2005, pp. 30-40). Geometry not only should not be averse to mechanics but should incorporate it without losing its characteristic of constituting exact knowledge:

> "It seems very clear to me that if (as is customary) we consider geometrical that which is precise and exact and mechanical that which is not, and if we consider geometry as the science that furnishes a general knowledge of the measures of all bodies, we have no more right to exclude the more composite lines than the simpler ones, provided that one can imagine them as described by a continuous motion or by several motions that follow each other, and of which the last ones are completely regulated by those that precede" (Descartes, AT, VI, p. 389).

If the curve is generated by a continuous movement or by several movements that follow one another, the latter being complementarily determined by the preceding ones, then it must be considered geometric, that is, exact (Bos, 2001, p. 409). Only when this continuous and causal connection between the preceding and subsequent movements is broken, should the line generated be designated as inexact. Roughly speaking, the system that allows the drawing of more complex lines while preserving the geometric nature of the curve is identical to the operating mechanism of a machine, not just any machine, but an automaton. And is enough for us to imagine this mechanism: it does not need to have passed from the idea to the physical world.

The relationship between geometry and autonomous machines, framed through the concept of exactness, facilitates the mechanization of geometry and the geometrization of mechanics. This interplay allows one structure to be translated into the other. However, the autonomous mechanism corresponding to a specific exact curve can be difficult to visualize. In some cases, the challenge is so considerable that constructing such a machine becomes impractical. In other words, there is no consistent or reliable method for translating between geometric and mechanical instances. We will explore how a similar translation method exists between geometry and algebra.

Imbued with the machine metaphor, Descartes reconceives geometry, nature, and man. He distinguishes between mechanical machines, which he calls artificial machines, and biological organs and living beings, which he refers to as natural machines. The former are self-moving and ubiquitous, found in fountains, clocks, mills, and other devices (Descartes, AT, X, pp. 231-2; XI, pp. 120, 130-1, 204, 505). As the renowned essayist Thomas Carlyle later observed, modernity is the age of machines (Carlyle, 2023). However, Descartes identifies two distinct values in these instruments. The first is their utilitarian value: machines, as tangible tools, serve as means to achieve precise objectives. The second is their epistemological or heuristic value, particularly as conceptual tools that aid in illustrating the properties and operational definitions of regular curves in geometry. More broadly, they serve to model various aspects of mathematics, nature, and even the human mind (Burnett, 2005, pp. 33-4).

For instance, constructing a circle with a compass can be described as an algorithm consisting of the following steps: i) one end of the compass must be fixed; ii) the opening angle should be adjusted according to the desired radius; iii) the compass must be rotated around the fixed point without altering the opening angle. While the compass is essential for this task, creating the figure requires a precise sequence of instructions. If the steps are not properly organized, the result will deviate from the intended outcome. The propositions related to these instructions involve operations concerning the components, joints, and movements of the compass[2].

The operational definition constructs the circle, whereas its intentional definition – described as the set of points equidistant from a fixed point on a plane – articulates its essence by integrating the Aristotelian concepts of proximate genus and specific difference (Parry & Hacker, 1991, p. 86). It is crucial to distinguish between these two modes of defining a curve. The first approach, particularly significant to Descartes, offers a method that facilitates the construction of knowledge

2 The inclination to value machines is closely associated with the promotion of operational definitions. In addition to Descartes, Thomas Hobbes (1588-1679) also explores the mutual implication within this relationship. However, for the purposes of the present discussion, it suffices to focus on the conceptual dimension of this interplay. He offers several examples of operational definitions, which, in their original context, are more accurately referred to as generative definitions (Adam, 2019, p. 44). To underscore his position, Hobbes contrasts his approach with that of Euclid, criticizing the reliance on self-evident or common-sense definitions found in *Elements* (Bird, 1996, p. 225; Hobbes, 1992, p. 82). Hobbes identifies two distinct types of definitions: the first pertains to "words which mean things of which some causes can be understood", while the second concerns "words which mean things of which no causes can be understood" (Gauthier, 1997, pp. 513–4). The former, closely aligned with operational definitions, conceptualizes terms by articulating their causes or generative processes. This methodological framework is evident in both geometry and civil philosophy. For instance, in geometry, Hobbes defines a circle as "a figure formed by the circumduction of a straight line in a plane" (Hobbes, 1992, p. 81). Similarly, in civil philosophy, Hobbes asserts that injustice cannot exist in the state of nature, as "where there is no common power, there is no law; where there is no law, there is no injustice" (Hobbes, 2012, p. 196). Here, the concept of injustice is defined causally, rooted in the existence of law and the processes through which it is generated.

without necessitating a complete understanding of the underlying essence of the object. As such, these intentional definitions can be treated as theorems to be demonstrated, and the operation of the conceptual system or machine, with its systematic movements, can be interpreted as a method for deriving these definitions from more fundamental concepts[3]. The epistemological value of machines is thus a central aspect of the Cartesian project (Descartes, AT, X, p. 233).

4. The algebraization of geometry

At the outset of *Geometry*, Descartes formulated rules of conversion using a unit of measurement and Thales' theorem, thereby laying the foundation for analytical geometry. This innovation, among other contributions, redefined the criterion for classifying curves. Henceforth, the classification no longer depended on mechanical constructions but rather on representation through algebraic equations. As he explains regarding his objective in *Geometry*:

> "But, in order to comprehend together all the curved lines that exist in nature and to distinguish them orderly in certain genres, I have no better knowledge than to say that all the points of the lines which can be called geometric, that is, of those which admit some precise and exact measure, have necessarily some relation to all the points of a straight line, a relation that can be expressed by some equation, the same for all points" (Descartes, AT, VI, p. 392).

The superior nature of algebraic language, due to its greater heuristic capacity, is evident in geometry itself. By adopting algebraic language to address geometric problems – such as the Pappus problems – it becomes possible to develop algorithms through algebraic equations, providing criteria that are both more accurate and more general than those of ancient geometry. However, as previously noted, Descartes did not inherit the mechanical geometry of the ancients. Instead, he inherited a mechanics applied to geometry that lacked exactness and, in response, redefined it. His mechanized

3 Karl Raimund Popper (1902-1994) claims that essentialist obscurantism has played a negative role in modern science (Popper, 1995, p. 31). Although traces of this methodology can be found in authors such as Galileo and Descartes, some elements place them as partial critics of essentialism. Our reading of the Cartesian theory of knowledge as representation aims to move it away from methodological essentialism and closer to an instrumentalist epistemological attitude.

geometry became the foundation from which he derived the rigor and accuracy necessary to advance other fields. Understood in this way, science – not the trial-and-error methods of craftsmen – should serve as the guiding principle for the design and production of machines[4].

This discovery represents a significant innovation. By dissolving the boundaries between geometry, algebra, and arithmetic, Descartes developed a unified language that combines the heuristic strengths of each discipline. Traditionally, the product of two line segments was interpreted as an area, while the product of three segments was understood as a volume. Descartes, however, redefined these operations by introducing a unit of measurement and a coordinate system. Through this framework, arithmetic operations – addition, subtraction, multiplication, division, and root extraction – were applied to geometric segments, where the product (or division) of segments was interpreted not as an area or volume but as another segment (Descartes, AT, VI, pp. 369-75).

Through this innovation, Descartes demonstrated how to translate a geometric problem into an algebraic one, thereby enhancing the heuristic resources available for constructing algorithms. This advancement not only facilitated the representation of geometric problems but also provided new means for solving them (Descartes, AT, VI, pp. 376-81). In this context, the algebraic representation of geometry enables the development of algorithmic procedures, providing a level of control that surpasses both the mechanical methods of the ancients and Descartes' own mechanized geometry. Analytical geometry functions as an algebraic framework for geometry, laying the groundwork for the transition from analog to digital processes. This shift entails moving from the domain of geometric constructions and mechanisms to a focus on numbers and equations.

With this development, the need to work with physical representations, as was the practice in ancient geometry and Cartesian mechanized geometry – where instruments were used to generate geometric curves – becomes obsolete. Descartes addresses this shift in his *Geometry*, where the study of curves is no longer based on their reproduction through mechanical procedures. Instead, it is approached through the use of algebraic equations[5].

4 As a product of science, technologies such as autonomous machines – along with telescopes, microscopes, and similar devices – are more effective than their handcrafted counterparts (Gauvin, 2006). This increased effectiveness stems from the accuracy in design that science imparts to technology. Descartes views science – specifically, mechanized and analytical geometry – as essential to the advancement of technological production. His correspondence with Constantyn Huygens (1596-1687) exemplifies this notion: science plays a crucial role in fostering the prosperity and well-being of humanity (Descartes, AT, I, pp.330, 335-7, 433, 614, 761-6).

5 In the *Discourse on the Method*, two essays, *Dioptrics* and *Geometry*, were included to demonstrate how the method can be applied. Each book uses different forms of geometry to illustrate its points. *Dioptrics* focuses on mechanized geometry, as it aims to construct a hyperbolic lens polishing machine (Burnett, 2005, p. 59). Meanwhile, *Geometry* emphasizes analytical geometry to showcase its heuristic superiority as a new tool (Crippa, 2014). The representation of objects is the result of careful consideration influenced by the epistemic subject's purposes in a specific area of human knowledge (Landim Filho, 1992, p. 78).

Although autonomous machines can achieve accuracy, the conceptual framework in which they are conceived does not offer as many heuristic resources as algebraic representation. For example, a parabola can be constructed through a mechanical process or described by an algebraic equation (Descartes, AT, VI, pp. 369, 375, 387, 442). However, the algebraic representation facilitates the transformation of geometric problems into algorithms, such as the Bhaskara formula for finding roots. Ultimately, this approach enables the execution of mental experiments and controlled virtual simulations – tools of great value to physics.

The language used to describe nature no longer needs to be figurative or analogical to the objects it represents. Instead, anything that can be quantified can be expressed through algebraic representation. This shift enables access to a framework where processes can be more easily algorithmized, problem-solving capabilities are enhanced, and simulations become possible. As a result, the primary goal of a representational language is to introduce greater rationality into the problem-solving process. This approach also aligns with the concept of objectivity, as it establishes intersubjective procedures that can be universally accessed and reproduced.

In Cartesian epistemology, objectivity arises from a cognitive process that transforms entities of reality into representations within consciousness, which are capable of maintaining exactness. However, the truth or correspondence of these representations with external reality cannot be verified through direct comparison. Objectivity does not primarily concern the impartiality of the subject; rather, it is the subject that effectuates the objectification of reality. Instead, objectivity pertains to the precision with which deductions are made from fundamental concepts that are understood through clarity and distinctness (Landim Filho, 1992, pp. 37, 124).

5. The representation of nature by machine

One of the most notorious machine designs in Descartes' work is the hyperbolic lens polishing machine, mentioned in *Dioptrics* (Descartes, AT, VI, pp. 211-28). However, based on the previous discussion, it should be evident that machines serve a purpose beyond geometry and the transformation of raw materials. They represent the grand metaphor and model of nature itself. To support this, two key theses are required: an epistemological thesis that interprets ideas as representations, and an ontological thesis that reduces all material things to extension and motion.

Knowledge is essentially representation. Modern science distinguishes itself from ancient and medieval science by requiring the establishment of an objective domain of investigation, grounded in a framework of nature. The more precisely this framework is defined, the greater our ability to predict outcomes through calculations and to explore it through experimentation.

Conceiving an experiment is not an independent act that precedes theorization; rather, it is a consequence of theorization. This conception of the experiment is central to rationalism. For a rationalist, an experiment must first refer to the idea of experimentation, which encompasses not only physical experiments but also thought experiments, as well as mechanical, geometric, or algebraic simulations. To facilitate this, the foundational framework upon which mathematical and empirical science is built must be carefully chosen.

> "To set up an experiment means to represent or conceive the conditions under which a specific series of motions can be made susceptible of being followed in its necessary progression, i.e., of being controlled in advance by calculation. But the establishing of a law is accomplished with reference to the ground plan of the objectsphere. That ground plan furnishes a criterion and constrains the anticipatory representing of the conditions. Such representing in and through which the experiment begins is no random imagining" (Heidegger, 2002, pp. 75-85)???.

Descartes argues that for a specific sequence of movements to be considered necessary and thus predictable through calculation, it is crucial to conceive of a series of movements where each subsequent movement is entirely determined by the preceding ones. This concept resembles that of an autonomous machine. The domain explored by Descartes – the Cartesian *mathesis universalis* – is grounded in the model of the autonomous machine, which can be mathematized and anticipates the conditions necessary for experimentation.

If the issues at hand require exact answers, they must be represented in a manner that fulfills this requirement. To achieve this, Descartes suggests modeling various phenomena as a machine. This approach allows the dynamics of the phenomena to be understood geometrically, through mechanized geometry, and subsequently expressed algebraically as equations using analytical geometry.

This strategy is applied to both *res extensa* and *res cogitans*, as evidenced in Descartes' *Traité de l'Homme*, where he analyzes and represents various aspects of reality, including the human body and mind. The bodies of both animals and humans are reduced to articulated systems of parts (Descartes, AT, XI, pp. 120, 126, 141, 173, 197), based on the theory that complex systems, including biological ones, are composed of simple machines. These machines are all governed by the principle that allows a small force to lift a heavy weight[6]. Bodies, understood as machines, are made up of organs that function as gears or pulleys, similar to those found in a watch (Descartes, AT, IV, p. 408). The joints between organs are formed through contact or by means of chains and belts. There are multiple ways these organs can articulate, allowing the body to perform movements necessary to adapt to various circumstances. These articulations are naturally programmed and activated by sensory impressions (Descartes, AT, XI, pp. 67, 137-8).

In addition to the body, the human being has a rational soul, which constitutes the essence that makes him capable of programming with the use of language and constructing deductive reasoning (programs). In this characteristic lies the defining element that distinguishes man from animals (Descartes, AT, VI, p. 56; IX, pp. 14, 20, 42, 56-7, 229; XI, p. 131). Language is a crucial element of this distinction, particularly those that can support exactness, such as geometry, which according to Galileo (1564-1642), is not merely any language:

> "Philosophy is written in this grand book, the universe, which stands continually open before our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth" (Galilei, 1623).

However, for Galileo, nature is expressed in geometric terms, whereas for Descartes, human ingenuity represents it mechanically. Nature, in Descartes' view, is a machine, not from an ontological perspective, but from an epistemological one: "nature is not a source of standards but is itself subject to the higher standard of Cartesian rationality" (Ribe, 1997, p. 53). Our intellect models nature as an autonomous machine. To disregard this metaphor is not merely a failure of pragmatism; it is to forfeit the ability to think with certainty and exactness. As Descartes states, "We are not sufficiently accustomed to thinking of machines, and this has been the source of all error in philosophy" (Descartes, AT, V, p. 174).

6 Descartes developed a small treatise on machines, later sent to Huygens, in which he declares that any machine should always start with the simplest components – the pulley, the slope, the wedge, the cog-wheel, the screw, the lever – those that cannot be decomposed later (Descartes, AT, II, pp. 224-8).

This error undermines most rationalization attempts that claim to be scientific. The structure of reasoning can be articulated through two approaches: the method of analysis and the method of synthesis (Battisti, 2010). These methods can be linked to autonomous machines, depending on whether they are considered in terms of the order of conception, genesis, or discovery, or in terms of their operation and functioning. The synthesis method, which serves as a method of proof, contrasts with the analysis method, which is a method of discovery. The synthesis method is commonly employed in geometry to prove theorems from axioms and closely resembles the direct operation of machines. As a proof method, the order of reasoning should progress from the simple to the complex, and from the known to the unknown.

> “Order requires that the concepts introduced first must be understood without relying on those that come afterward. Additionally, the subsequent ideas should be arranged in such a way that they are explained solely by the ideas that precede them. I made a strong effort to adhere to this order in my Meditations” (Descartes, AT, IX, p. 121).

The comparison with the text that reclassifies mechanical curves (footnote 8) underscores the profound affinity between mechanical and cognitive processes, both of which are characterized by exactness. In this context, the method serves as a corrective for errors, demonstrating the mechanization of reasoning – human thought processes operate systematically, akin to machines. Mechanics and reasoning mirror each other, implying that autonomous machines can serve as a model for the correctness of the *ordre des raisons* (Battisti, 2010, p. 6).

> “Those long chains of reasoning, simple and easy as they are, of which geometricians make use in order to arrive at the most difficult demonstrations, had caused me to imagine that all those things which fall under the cognizance of man might very likely be mutually related in the same fashion” (Descartes, AT, VI, pp. 19-20).

A strong connection between individual elements of reasoning is essential to ensure the robustness of a logical chain. The "long chains of reasoning" must adhere to the principles of the Cartesian mechanism for generating geometric curves, which operates through "continuous motions that follow one another, with the latter fully determined by the preceding ones." In this framework, Descartes' logical chain reflects the transition from clause to clause via deduction, analogous to the transfer of motion from one component to another within an autonomous machine, without reliance on any external agent. The robustness of such a system depends not solely on the distinctiveness of individual components or propositions but also on their precise arrangement within the mechanism or the order of ideas. This arrangement ensures the accuracy and coherence of the mechanism or argument. Thus, there exists a profound correspondence between mechanical systems and the human mind when considered from the standpoint of precision and exactness.

From this perspective, the investigation of an unknown phenomenon aims to elucidate it by referencing a known autonomous mechanism. Within this framework, the concepts employed to explain the phenomenon are shaped by the principles of the underlying mechanism. As a result, the phenomenon is interpreted through the logic governing the operation of the machine, which is directed toward its programmed purpose – often described as the "spirit of the machine". Moreover, the concepts are defined operationally within this mechanistic framework. This operationalist perspective not only simplifies external phenomena by reducing them to autonomous mechanisms – thereby achieving the unified explanatory framework characteristic of modern science (Heidegger, 2002, p. 80) – but also reflects a ontological connection between the mind and the world, both of which are conceived as mechanistic systems.

The distinction between mind and body occupies a central position in Descartes' philosophy. By employing methodical doubt, this separation enables the rejection of sensory illusions, childhood prejudices, and arbitrary constructs of the will. It further establishes the mind as the domain of pure intellect, capable of apprehending *a priori* the divine laws governing natural phenomena[7]. Yet, despite this distinction, both mind and body are conceived as mechanized entities, integrated into the precise plan of nature.

Building on mechanical curves, the Cartesian framework extends its objective domain of investigation to encompass natural phenomena, animals, and human beings (Heidegger, 2002, p. 74). This approach reductively aligns these domains with the operations of autonomous machines, forming a foundational plane of exactitude. The translation of mechanical principles into geometry, and subsequently into algebra, enhances the epistemological significance of this foundational plane. This progression incorporates increasingly powerful heuristic tools for representing natural phenomena and solving complex problems.

The Cartesian project presents the autonomous machine as a benchmark for understanding knowable reality, thereby linking the representation of reality with the process of objectifying mechanics – remember that, for the ancients, mechanical curves were not exact. This framework

7 Without this separation, the only recourse would be the empiricist approach of inferring these laws *a posteriori* from their observable effects (Descartes, AT, I, pp. 250-2).

extends to bodies, the mind, and natural phenomena. The knowledge that operates within this domain critically examines these realities, questioning how and to what extent they are amenable to representation through a structure devised by reason and characterized by exactness (Heidegger, 2002, pp. 75, 79, 85).

Since the representation of reality must be modeled after the machine, all concepts applied to autonomous machines equally extend to representations. In doing so, Descartes – although not using identical terminology – establishes a framework in which we can identify elements that resonate with the digital age[8]. In the description, "one can imagine them [curves] as described by a continuous motion or by several motions that follow each other, and of which the last ones are completely regulated by those that precede", we can intuit the very nature of the machine, expressed through a language (whether mechanical, geometric, or algebraic) and executing a program.

A machine, in its general form, consists of components that are interconnected through gears, belts, straps, chains, and similar mechanisms. The concepts related to these components, their interconnections, the sources of movement, the various laws that describe the conservation of movement, and the intended purposes of machines collectively form a conceptual framework, or language. The tools within this framework that enable problem formulation and resolution are referred to as heuristic resources. These parts must move relative to one another with high precision, transmitting the initial movement through the mechanisms in accordance with the machine's design. The arrangement of connections and the transmission of movement, governed by this design, constitutes the rule, law, program, or algorithm that dictates how movement is controlled and transmitted[9].

The representation can be understood as comprising two components: language and program. The exact representation of reality does not imply a faithful reproduction of the object or phenomenon, but rather that the chosen representation is capable of generating and preserving exactness. The autonomous machine begins by receiving an initial movement, which is transmitted through several stages to a final result, without interference in any of its intermediate processes. It is this specific mechanical relationship between cause (the initial movement) and effect (the resulting output) that constitutes exactness. Similarly, reasoning can be characterized in this way, where the

8 For instance, Herert A. Simon (1916-2001) identifies two interdependent factors in the representation of problems: the formalization of a coherent conceptual framework and the development of problem-solving programs (Simon, 1977, pp. 224-6).

9 Control over the transmission of movement is physically enacted by mechanisms. However, both mechanized geometry and analytical geometry enable the representation of this control, with movement now understood in a broader sense, through geometry and algebra (Descartes, AT, III, p. 55; XI, p. 120). Thus, depending on the language used to represent a system, programs (or laws) can be articulated in different ways: mechanically, as seen in the principle of levers; geometrically, as in the various Gibbs laws of thermodynamics for homogeneous substances; or algebraically, as demonstrated in numerous instances in physics.

precision of deductions that connect clear and distinct intuitions to theorems or conclusions within a theoretical framework provides certainty to the truths derived from reason (Landim Filho, 1995, p. 125). Consequently, exact representations position science as a body of knowledge that is certain, stable, and true.

At the intersection where truth, certainty, and exactness often become indistinguishable (Biasoli, 2023, p. 7), a representation is deemed true if the corresponding ideas can be seamlessly integrated into a structure that conveys the certainty of the truth of fundamental concepts across the entire theoretical system. This notion of representation fosters a uniformity between the object and the modality of science – one that assumes an inherent imbalance in their relationship when objects are situated within the frameworks of mathematical science. As a result, truth is reduced to the psychological experience of certainty[10]. The actual system of science consists of this combination of processes (representation) and attitudes (operationalism) that facilitate the objectification and accessibility of reality (Heidegger, 2002, pp. 82-4).

6. The mechanization of heat science

The 18th century is the century of the steam machine, developed and applied mainly in England. Despite James Watt's (1736-1819) great efforts to study the subject, progress in this area was made, in large part, by trial and error. At the same time, the abundance of coal in England did not stimulate the theoretical framework of this topic according to the perspective of scarcity, which contributes to making unnecessary an approach around the problem of raising the efficiency or yield of this equipment.

10 The alignment between Descartes and Hobbes, as discussed earlier, demonstrates how the challenge of establishing criteria for truth – fundamental to guiding knowledge production during the early modern period – is methodologically linked to the processes of knowledge dissemination. For Descartes, this meant anchoring truth in the clarity and distinctness of ideas, which were transmitted through a theoretical framework based on the "cascade of truth" model (Chiappin, 1996, pp. 203-6). However, with the advancement of science, defending the validity of these criteria has become increasingly challenging. Thinkers such as Duhem and Popper have pointed out the limitations of the cascade model, along with the redefinition of the knowledge base as hypothetical, prompting the emergence of alternative models of knowledge dynamics, including those focused on convergence toward truth and instrumentalist approaches. As a result, particularly in the natural sciences, attention has shifted from criteria for establishing the foundational truth to the mechanisms governing the transfer of knowledge within theoretical systems. In this context, Hobbes's operationalism, which posits that truth must be constructed through operational means, has gained renewed relevance: "it takes the truth of geometric definitions to be established by our power to generate the definiendum through the process outlined in the definiens" (Gauthier, 1997, p. 514).

With the end of the war between England and France and the re-establishment of exchange between the two countries, there has been a period of economic prosperity and a new interest in thermal machines in France. However, in this country, where coal is far less abundant, the issue of efficiency becomes pressing. Unlike England, the machine to be studied and improved in France is the so-called high-pressure thermal machine, whose coal consumption is significantly lower (Dockinson, 2011; Fox, 1976, pp. 166-7). Nicolás Léonard Sadi Carnot (1796-1832), a young military engineer who published in 1824 a pamphlet entitled *Réflexions sur la puissance motrice du fue et sur les machines propres à développer cette puissance* – henceforth only *Reflexions* – is aware of these conditions and the great benefit that thermal machines can bring to mankind (Carnot, 1897, p. 37).

Despite extensive application in Carnot's time, steam machines had not yet developed their full potential. The revolution was announced, but it was not yet a reality. Without this evolution, their potential will not be unleashed, and progress will be halted. Carnot then makes a bold judgment:

> "Notwithstanding the work of all kinds done by steam engines, notwithstanding the satisfactory condition to which they have been brought today, their theory is very little understood, and the attempts to improve them are still directed almost by chance" (Carnot, 1897, p. 42).

This judge would be temerary if he were not aware of the reality quite different from another field of physics. While the theory of steam machines was at an early stage, the mechanical machine theory was fully established. And he knew this very well. In fact, his father, Lazare Nicolas Marguerite Carnot (1753-1823), was one of those who contributed most to the formalization of a general theory of mechanical machines. The discrepancy between these two theoretical fields, in the young scientist's eyes, could not be greater.

A general theory for heat engines did not exist. Since these engines operate based on the action of heat, such a theory could only be developed once the underlying physical laws governing the effects of heat were sufficiently understood and generalized. According to Sadi Carnot, the construction of thermal machines is a blind endeavor without a thorough understanding of the principles that govern this domain. This assertion is an echo of Leonardo da Vinci's (1452-1519) cautionary advice, which encapsulated the spirit of modern scientific inquiry: "Those who fall in love with practice without science are like a sailor who enters a ship without a helm or a compass, and who never can be certain whither he is going" (Richter, 2012, p. 456).

The reason for Carnot's dissatisfaction with the theory of the steam engine was its lack of universality. The conclusions were linked to particular machines or processes, where the phenomenon of heat movement was not considered from a general point of view. Thus, it became very difficult to discern the principles and laws of this phenomenon. Carnot, then, recommends examining in the most general way the principle of the production of motion by heat, independently of any mechanism or any particular agent, namely:

> "Wherever there exists a difference of temperature, motive power can be produced. Reciprocally, wherever we can consume this power, it is possible to produce a difference of temperature; it is possible to occasion destruction of equilibrium in the caloric" (Carnot, 1897, p. 51).

The recommendation that "it must be considered independently of any mechanism or particular agent" should not be interpreted as a rejection of the mechanism itself, but rather as an emphasis on the generality of the principle that "whenever a temperature difference exists, motive power can be generated". The phenomenon of producing motive power from heat motion should, therefore, be described in terms of a simple and general heat engine (Carnot, 1897, p. 43). At the time, heat was primarily regarded as a subtle and indestructible substance, known as caloric. This understanding shaped the conceptual framework for the operation of thermal machines. For Carnot, heat engines were seen as devices that carried caloric, transferring heat from a source to a reservoir maintained at different temperatures. It is within this context that Carnot's assertion, "the production of motive power is then due not to the actual consumption of caloric, but to its transportation from a warm body to a cold body" (Carnot, 1897, p. 46), should be interpreted[11]. According to this view, the machine must absorb heat from the hot source and transmit it to the cold reservoir through an intermediate substance.

Carnot's purpose, however, is not limited to describing the operation of thermal machines. He seeks to maximize the production of driving force. For this, another condition is required: "In the bodies employed to realize the motive power of heat, there should not occur any change of temperature, which may not be due to a change of volume" (Carnot, 1897, p. 57). This condition, inspired by Watt's expansion law (Fox, 1969), immediately becomes a principle.

11 In this article, power, motive power, driving force, and mechanical effect are all similar expressions of what is today understood as mechanical work.

> “This principle should never be lost sight of in the construction of heat engines; it is its fundamental basis. If it cannot be strictly observed, it should at least be departed from as little as possible” (Carnot, 1897, p. 57).

The sequence of processes that leads to the maximum production of motive power must, therefore, adhere to the condition that all caloric transfer occurs without a change in temperature. Additionally, any temperature variation should only result from changes in volume. Deviations from this ideal would result in energy losses when restoring equilibrium within the caloric system – an outcome to be avoided, although not entirely achievable in practice. The relationship between temperature and volume variations makes it possible to conceptualize a heat engine as a mechanical machine and to reduce a thermodynamic system to a exact mechanism in the Cartesian sense: only under this condition does the heat engine operate reversibly. This provides a scientific principle that should guide the construction of the thermal machine, one that is defined solely by a thermodynamic framework geometrized through the exact equivalence of temperature and volume variations.

The rationalization introduced by Carnot is grounded in an analogy that reflects the Cartesian strategy of reducing the unknown to the known. The heat engine, a novel concept, is ideally modeled as a mechanical device, specifically as a water wheel that generates motive power from the flow of water. In a hydraulic engine, the motive power generated is contingent upon the volume of water and the height of the fall. Similarly, in the heat engine, the production of motive power depends on the quantity of caloric and the temperature differential between the heat source and the reservoir[12].

The water wheel, as a mechanism for transferring motion, provides a framework for theoretical refinement wherein, under ideal conditions, the entirety of the water's movement is harnessed by the wheel. This mechanical device, in theory, can be optimized to achieve a yield of one hundred percent. However, its thermodynamic counterpart does not attain such perfection, even in idealized scenarios. The thermal engine operates based on the flow of heat (caloric), and the generation of driving force depends on the transfer of heat between a hot and a cold reservoir. Once the mechanism is established between these temperature differences, the engine operates autonomously.

12 According to this analogy, caloric functions similarly to water, with its quantity being conserved. In contrast, within the energy paradigm, heat is not regarded as a substance but rather as an intrinsic property of matter, capable of being transferred or transformed.

Carnot's analogy, which underpins this concept, reflects his adherence to the caloric theory of heat. In this framework, caloric is treated as a conserved substance, analogous to water, that cannot be transformed into another form[13]. Notably, it is the principle derived from this analogy that establishes thermodynamics as a distinct domain of physics, irreducible to purely mechanical principles (Erlichson, 1999).

The study of the efficiency of mechanical machines was a project initiated by Lazare Carnot, the father of Sadi Carnot. In this context, *Réflexions* can be interpreted as the outcome of a collaborative scientific endeavor between father and son. Lazare Carnot developed a comprehensive theory of machines, building upon the foundational work of Jean le Rond d'Alembert (1717–1783) and Christiaan Huygens (1629–1695), who themselves were influenced by Descartes. This approach differs from Newtonian science, which represents an alternative paradigm in the study of mechanics (Nage, 1961, p. 200; Pisano, Coopersmith & Peake, 2021, p. xlviii).

In this context, we refrain from employing the conceptual and methodological framework of Newtonian science, opting instead for an approach specific to the science of machines. This perspective is grounded in the fundamental principle that mechanical machines generate mechanical work by transmitting motion from one body to another: "Mechanics are nothing else than the theory of the laws of the communications of the motions" (Carnot, 1803, p. xiii). This principle operates within the constraints of the impossibility of perpetual motion and the independence of a machine's efficiency from the nature of the working substance (Pisano, Coopersmith & Peake, 2021, p. lxxix).

Rooted in the Cartesian approach, the aim of a "theory of the laws of motion communication" extends beyond merely explaining the mechanisms enabling such communication; it also emphasizes the efficiency and precision of this process. Lazare Carnot's work aligns with the Cartesian project, as does the thermodynamics later pioneered by his son, Sadi Carnot. Thus, while this branch of science develops independently of Newtonian mechanics, it positions itself as a continuation and extension of the Cartesian framework (Pisano, Coopersmith & Peake, 2021, p. lxxv).

The analogy with the water wheel enables the theory of thermal machines to integrate a fundamental concept from mechanics, which is essential for maximizing the production of driving force. By reversing the operation of a water wheel, water can be returned to its original height; similarly, by performing work on a heat engine, caloric can be returned to the heat source. This process requires that the transfer of caloric be tied to the variation of an extensive property – in this

13 The result of applying the condition that the temperature fluctuation must result from the volume fluctuation so that there is no useless heat transport as far as the generation of motive power is concerned is that no heat engine has a higher efficiency than the reversible heat engine. Later, as part of Clausius' reformulation, this result was incorporated into the so-called second law of thermodynamics (Clausius, 1879, pp. 80-7; Newburgh, 2009, p. 713).

case, the volume occupied by the working substance. For substances employed in generating motive power from heat, changes in temperature occur solely as a result of changes in volume. Under these ideal conditions, it becomes possible to manipulate one of these variables (temperature or volume) by using the other as a control parameter.

Thus, the transport of caloric, which induces a change in the volume of the working substance, can be uniquely reversed. In an equivalent manner, the reverse variation in volume generates the transport of caloric in the opposite direction. Reversibility, however, is not simply the reversal of the operation itself; it also requires the restoration of the initial values of the state functions.

The thermal machine described by Carnot is ideal and not practically achievable, a fact of which Carnot was fully aware. Descartes, on the other hand, did not share this awareness. While his curve-drawing machines could achieve the desired results, his hyperbolic lens polishing machine proved impractical, leading to frustration and a strain in his relationship with his trusted craftsman, Jean Ferrier (Gauvin, 2006, pp. 188, 199). Carnot's recognition of the limitations of practical implementation further underscores the heuristic value of his machine. Indeed, the central argument concerning the efficiency of his machine is logically constructed. By coupling two reversible machines – the first generating motive power from the transport of caloric, and the second utilizing this motive power to return caloric to the heat source – it becomes possible, in an ideal scenario, to realize a perpetual motion machine. In this setup, neither external motive power is generated nor caloric accumulates in the reservoir. Consequently, while the efficiency of a reversible heat engine depends on the nature of the working fluid, it could theoretically be enhanced by employing a more efficient fluid. However, such a notion is ultimately deemed unacceptable.

> "This would be not only perpetual motion but an unlimited creation of motive power without consumption either of caloric or any other agent whatever. Such a conception is entirely contrary to ideas now accepted, to the laws of mechanics and sound physics. It is inadmissible. We should then conclude that the maximum of motive power resulting from the employment of steam is also the maximum of motive power realizable by any means whatever" (Carnot, 1897, p. 55).

This is an argument for reduction to absurdity, usual in mathematics and philosophy; namely, a statement is correct if its contrary leads to physically absurd situations. Therefore, the mechanical analogy is complete: thermal machines are reversible, as they rely on the precise relationship between volume and temperature, and their efficiency remains independent of the working substance. Carnot cannot implement his ideal machine without resorting to a theory of heat, in this case, the caloric theory. Mathematically, the caloric theory enables the identification of heat as a property, thus characterizing it as a state function (Lervig, 1976). However, the translation of this and other thermodynamic properties into mathematical language was undertaken by scholars such as Emile Clapeyron (1799-1864), Rudolf Clausius (1822-1888), William Thomson (1824-1907), and James Clerk Maxwell (1831-1879).

Autonomous machines are capable of generating exactness and, as a result, of constructing curves and an entire geometry that is equally exact. In the context of thermodynamics, the water wheel functions not only as a prototype for understanding the role of caloric but, in conjunction with the caloric hypothesis, establishes the exact equivalence between variations in temperature and volume, thereby creating a mechanical representation of thermodynamics. By mechanical representation, we do not merely refer to a description of the heat engine mechanism. In conceptualizing a mechanized heat engine, Carnot incorporates heuristic features of mechanical machines – such as reversibility and the role of the working substance – which together enable him to design a heat engine with the highest achievable efficiency. Carnot thus constructs a conceptual framework (language) endowed with sufficient heuristic power to establish reversibility (program) as the defining characteristic of a heat engine operating at maximum efficiency, irrespective of the working substance involved.

7. The evolution of heat science: the representations of thermodynamics

Like Descartes, the Carnot machine corresponds to a project built on the principle that any temperature variation must result from a corresponding volume variation, articulated according to the model of the water wheel, under the prohibition of the unlimited creation of driving force. It translates the condition of maximum power production and imposes the independence of this result from the nature of the working fluid. In this way, the Carnot engine, which is an autonomous machine – due to the circularity of its operation once the temperature difference between the heat sources has been established – designed by science and not by trial and error, assumes the role of an operational definition of the most efficient heat engine.

The language that Carnot employs incorporates machine components or engineering concepts (furnaces, boilers, steam cylinders, pistons, caloric, driving force, etc.), experimental laws (volume variation is associated with the temperature variation of a gas, characteristics of isothermic and adiabatic processes, the driving force operated by gas is greater in expansion than in contraction, etc.), and ontological assumptions (the nature of heat, unlimited production of the driving force is unacceptable, etc.). With this language, a program is designed, a series of steps whose realization results in the maximum possible production of driving force from the transport of a certain amount of caloric.

Carnot is aware that the program he develops is not the only one to achieve this goal. Any program that corresponds to a reversible cyclic sequence of thermodynamic processes also performs it. Their machine, therefore, is a particular case of a set of thermal machines that maximize the production of driving force. Although Carnot's machine is particular, the analysis of the program it runs has allowed its author to infer the necessary and sufficient condition (reversibility) that leads to the maximization of the production of the driving force in general. The Carnot engine has become the operational definition of a heat engine because it has historically been the most widely used motive power maximizing algorithm.

Carnot's original problem is solved in a representation in which the language is the language of (mechanical and steam) machines, and the algorithm (program) written to solve it, although particular, indicates the key to the universalization of that answer. However, this idea of a machine is impractical. This representation constitutes a conceptual space with heuristic power in which concepts, laws, and metaphysics are employed to describe the operation of thermal machines. The analogy with the water wheel adds the science of mechanical machines to the incipient science of heat engine. The heuristic power of the representation formulated by Carnot is no longer that original to thermal machines. It was enriched.

Something similar happened with geometry and algebra in the synthesis produced by Descartes, in which the heuristic resources of these spaces are combined to form analytical geometry. Carnot knows that his project needs proper representation. Although the problem can be articulated within the framework of steam engine engineering of the time (Mendoza, 1976), its resolution – particularly from the perspective of maximizing efficiency – necessitates a representation with enhanced heuristic capabilities. This improvement is realized through the incorporation of methodologies and principles from the science of mechanical machines. The result of this assimilation is that the language for solving the problem is improved; it becomes logically more consistent, enabling not only the construction of a program that solves the problem but potentially a series of equally efficient programs (Lucena, Laranjeiras & Chiappin, 2019).

Although the representation introduced by Carnot lacks intrinsic mathematical tools, unlike subsequent thermodynamic frameworks, the concept of reversibility enables more controlled simulations of candidate machines designed to maximize driving force. With this general principle, programs with such characteristics proliferate, allowing cyclic thermodynamic processes to be governed by reversibility rather than trial and error.

Carnot's effort can be understood as the construction of a representation in which the concept of the reversible thermal machine could be fully realized. From this perspective, the Carnot engine represents the culmination of this theoretical framework. The analogy between the thermal machine and the water wheel, crucial to the development of this conceptual space, situates Carnot's theory within what can be described as a mechanical representation. However, in the current context, this designation appears redundant and may be considered superfluous. In subsequent discussions, particularly in examining Clapeyron's contribution, we observe the abstraction of the Carnot engine from its mechanical components, reducing it to the cyclic sequence of thermodynamic processes imposed on the working substance, which are then represented diagrammatically. The theory's focus on physical machines with tangible parts and mechanisms, as opposed to diagrammatic abstractions, further justifies the application of this nomenclature (Lucena, Laranjeiras & Chiappin, 2023).

In summary, the preceding discussion can be encapsulated by stating that Carnot's theory is articulated within a mechanical representation, utilizing a machine-based language. Within this framework, through the application of logical and semantic resources, algorithms can be devised to address the problem of maximizing the driving force of thermal machines. The epistemological structure of the Cartesian project, as outlined at the outset of this work, is thus evident in Carnot's theory and the broader field of thermodynamics, with its multiple representations. This text, however, focuses exclusively on reconstructing the initial representation: the mechanical framework of thermodynamics.

In 1834, following the death of Sadi Carnot, Clapeyron sought to mathematize the theory of the ideal machine by abstracting it from its mechanical elements (Clapeyron, 1837). What remained was the thermodynamic cycle composed of two isothermal and two adiabatic processes, represented diagrammatically. This approach allowed Clapeyron to derive a mathematical expression for the efficiency of the ideal machine, which was absent in Carnot's original formulation. Additionally, his work expanded the phenomenological scope of the theory, albeit in a primarily qualitative manner (Lucena & Chiappin, 2017).

From 1834 to 1876, as new conceptual challenges emerged or previously unaddressed phenomena were incorporated into the science of heat, these issues were frequently approached through diagrammatic representations. Such representations were employed to facilitate problem-solving or to provide deeper insight into the phenomena. After this period we see the emergence of the geometric representation, which subsequently evolved into the algebraic representation, both developed by Josiah Willard Gibbs (1839-1903). These developments reflect a broader historical trajectory of representations in thermodynamics, wherein each new representation arose as a heuristic tool to address specific problems. However, as new challenges surfaced, older representations were often supplanted by more effective frameworks. We posit that this iterative process culminated in the formulation of potential thermodynamics for heterogeneous substances (Lucena & Chiappin, 2017, p. 302).

The evolution of thermodynamics can be seen as, in parallel with conceptual changes, a succession of representations with ever greater (mathematical and conceptual) heuristic power – ever more comprehensive representations – from Carnot's mechanical representation. To elucidate thermodynamics with the Cartesian epistemological structure, it ends up providing a methodology of rational reconstruction, possibly extensible to other sectors of Physics, in the form of Imre Lakatos's (1922-1974) research program methodology, but that, instead of valuing the formation of the hard core, irrefutable by methodological decision, appreciates more that problems are formulated and eventually solved in a certain representation. As an important part of the resolution of a problem is how to present it, in the case of persistent problems, a change in representation, accompanied by a conceptual adaptation, can greatly facilitate their resolution. Our reconstruction introduces a nuance to Lakatos' proposal. We will leave this analysis for another occasion.

8. Conclusion

In an effort to draw connections between Descartes and thermodynamics, a parallel can be observed between the evolution of geometric representations – progressing from mechanical curves, which are not exact, to mechanized geometry, and ultimately to analytical geometry – and the successive representations within thermodynamics. This succession is driven by the need to incorporate increasingly complex elements, achieved by transforming the original conceptual framework and constructing a new one with enhanced heuristic resources. A new representation in this sequence does not emerge *ex nihilo* but arises only after identifying the limitations or epistemological obstacles inherent in the previous representation (Bachelard, 2002, p. 24). The evaluation of a representation's heuristic power does not occur in isolation; it is the confrontation with concrete problems that compels scientists to explore theses boundaries.

For example, although mechanized geometry implements the accuracy of curves constructed by means of machines, the algebraic expression of these curves requires a sketch of concepts and their relations that constitute an alternative representation of geometry – namely, analytical geometry – with its own heuristic resources and greater power than previous versions of the discipline. In a simplified way, the geometry of the ancients and mechanized geometry are inadequate for the algebraic expression of mathematical objects. Similarly, the mechanical representation of thermodynamics is insufficient for the mathematization of Carnot's theory. Likewise, diagrammatic representations fail to account for Thomas Andrews' critical points, while geometric representations cannot accommodate the conformal dimensions required for modeling chemical reactions.

These representations, while exhibiting parallels to the mathematical frameworks outlined by Descartes, are intrinsic to the formulation and inherent development of thermodynamics itself. They are not merely external models or interpretations imposed upon thermodynamics. Furthermore, the resources provided by these representations extend beyond mathematics. For instance, the operability of the concept of reversibility exemplifies a non-mathematical resource. The geometric representation developed by Gibbs employs numerous non-mathematical tools to identify phase transition regions, the triple point, and the critical point of homogeneous substances.

This sequence of representations manifests, at each stage, in theories that incorporate heuristic resources and an expanded phenomenological domain relative to their predecessors. The transition from one representation to another, in alignment with a central thesis of Pierre Duhem (1861-1916), is linked to the efforts of scientists to reconcile disparate or conflicting theories, ultimately striving toward a unified theory that serves as a natural classification of physical laws (Duhem, 1981, p. 104). Consequently, the dynamics of scientific progress tend toward the development of increasingly comprehensive and abstract theoretical structures: "these efforts through slow and continuous progress have contributed to fusing together pieces of theory, which were isolated at first, in order to produce an increasingly unified and ampler theory" (Duhem, 1981, p. 295). The comprehensive and abstract framework that facilitates the unification of diverse elements within the science of heat – such as thermal machines, phase transitions, material properties, critical points, and chemical reactions – is the Gibbs algebraic representation. This representation operationalizes differential geometry applied to hypersurfaces, thereby enhancing the breadth and coherence of the field.

In the preceding paragraph, the introduction of Duhem into the discussion inadvertently establishes a connection – albeit an indirect one – with Descartes. This connection is notable because Descartes' radical mechanism, rooted in the dogmatic assertion that physics must be derived from metaphysics, represents one of the scientific doctrines most vehemently opposed by Duhem (Duhem, 1981, pp. 14, 43, 115, 276, 278). However, this convergence between the two thinkers does not occur within the domains of ontology or epistemology. Rather, it is situated in the realm of axiology, specifically in their shared emphasis on the heuristic enrichment of scientific theories. Two of the formalisms most extensively employed by Duhem – Hamiltonian formalism and Gibbsian thermodynamics – originate from principles grounded in mechanical systems: the lever and the Carnot heat engine, respectively. In this context, the link between Descartes and Duhem can be understood through their engagement with machines, albeit expressed in distinct ways by each thinker.

Although this structure highlights the dynamics of theorization through increasingly powerful representations, each representation must be sufficiently valued. For the historian of science, each representation shows the scope and the limits of science at every moment. Returning to the inaugural moment, the Carnot ideal machine meant the upper limit of the engineering of steam machines and, as an emerging property, it provides a substrate for the development of thermodynamics over the next 50 years of history. Clapeyron, William and James Thomson (1822-1892), Clausius, Maxwell, Gibbs, and others extract different aspects of it, giving a rush to conceptual reinterpretations, experimental laws, and fundamental laws. From this source emerges much more than a machine project. Carnot engine, instead of being merely a tool for generating motive power by heat, is reconceived precisely as an ideal system that generates our ideas.

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Declaration of generative AI and AI-assisted technologies in the writing process.

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